

Mark Scheme (Final)

January 2008

GCE

GCE Mathematics (6664/01)

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Principal for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \dots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \dots$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm p)^2 \pm q \pm c$, $p \neq 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

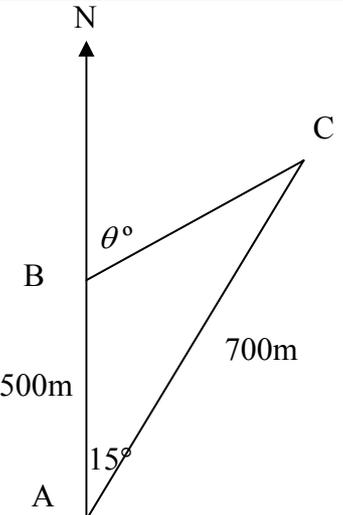
Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

2.	<p>(a) Complete method, using terms of form ar^k, to find r [e.g. Dividing $ar^6 = 80$ by $ar^3 = 10$ to find r; $r^6 - r^3 = 8$ is M0] $r = 2$</p> <p>(b) Complete method for finding a [e.g. Substituting value for r into equation of form $ar^k = 10$ or 80 and finding a value for a.] $(8a = 10) \quad a = \frac{5}{4} = 1\frac{1}{4}$ (equivalent single fraction or 1.25)</p> <p>(c) Substituting their values of a and r into correct formula for sum. $S = \frac{a(r^n - 1)}{r - 1} = \frac{5}{4}(2^{20} - 1)$ (= 1310718.75) 1 310 719 (only this)</p>	<p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>A1 (2) [6]</p>
Notes:	<p>(a) M1: Condone errors in powers, e.g. $ar^4 = 10$ and/or $ar^7 = 80$, A1: For $r = 2$, allow even if $ar^4 = 10$ and $ar^7 = 80$ used (just these) (M mark can be implied from numerical work, if used correctly)</p> <p>(b) M1: Allow for numerical approach: e.g. $\frac{10}{r_c^3} \leftarrow \frac{10}{r_c^2} \leftarrow \frac{10}{r_c} \leftarrow 10$</p> <p>In (a) and (b) correct answer, with no working, allow both marks.</p> <p>(c) Attempt 20 terms of series and add is M1 (correct last term 655360) If formula not quoted, errors in applying their a and/or r is M0 Allow full marks for correct answer with no working seen.</p>	
3.	<p>(a) $\left(1 + \frac{1}{2}x\right)^{10} = 1 + \frac{\binom{10}{1}\left(\frac{1}{2}x\right) + \binom{10}{2}\left(\frac{1}{2}x\right)^2 + \binom{10}{3}\left(\frac{1}{2}x\right)^3}{\underline{\hspace{10em}}}$</p> <p>$= 1 + 5x; + \frac{45}{4}(\text{or } 11.25)x^2 + 15x^3$ (coeffs need to be these, i.e, simplified)</p> <p>[Allow A1A0, if totally correct with unsimplified, single fraction coefficients]</p> <p>(b) $(1 + \frac{1}{2} \times 0.01)^{10} = 1 + 5(0.01) + \left(\frac{45}{4} \text{ or } 11.25\right)(0.01)^2 + 15(0.01)^3$</p> <p>$= 1 + 0.05 + 0.001125 + 0.000015$</p> <p>$= 1.05114 \quad \text{cao}$</p>	<p>M1 A1</p> <p>A1; A1 (4)</p> <p>M1 A1√</p> <p>A1 (3) [7]</p>
Notes:	<p>(a) For M1 first A1: Consider underlined expression only. M1 Requires correct structure for at least two of the three terms: (i) Must be attempt at binomial coefficients. [Be generous :allow all notations e.g. ${}^{10}C_2$, even $\left(\frac{10}{2}\right)$; allow “slips”.]</p> <p>(ii) Must have increasing powers of x, (iii) May be listed, need not be added; <i>this applies for all marks.</i></p> <p>First A1: Requires all three correct terms but need not be simplified, allow 1^{10} etc, ${}^{10}C_2$ etc, and condone omission of brackets around powers of $\frac{1}{2}x$ Second A1: Consider as B1: 1 + 5x can score A1 on Epen, even after M0</p> <p>(b) For M1: Substituting their (0.01) into their (a) result [0.1, 0.001, 0.25, 0.025, 0.0025 acceptable but not 0.005 or 1.005] First A1 (f.t.): Substitution of (0.01) into their 4 termed expression in (a) Answer with no working scores no marks (calculator gives this answer)</p>	

<p>4. (a)</p> <p>(b)</p>	$3 \sin^2 \theta - 2 \cos^2 \theta = 1$ $3 \sin^2 \theta - 2(1 - \sin^2 \theta) = 1 \quad (\text{M1: Use of } \sin^2 \theta + \cos^2 \theta = 1)$ $3 \sin^2 \theta - 2 + 2 \sin^2 \theta = 1$ $5 \sin^2 \theta = 3 \quad \text{cso} \quad \text{AG}$ $\sin^2 \theta = \frac{3}{5}, \text{ so } \sin \theta = (\pm)\sqrt{0.6}$ <p>Attempt to solve both $\sin \theta = +..$ and $\sin \theta = -...$ (may be implied by later work)</p> $\theta = 50.7685^\circ \quad \text{awrt } \theta = 50.8^\circ \quad (\text{dependent on first M1 only})$ $\theta (= 180^\circ - 50.7685^\circ); = 129.23\dots^\circ \quad \text{awrt } 129.2^\circ$ <p>[f.t. dependent on first M and 3rd M]</p> $\sin \theta = -\sqrt{0.6}$ $\theta = 230.785^\circ \text{ and } 309.23152^\circ \quad \text{awrt } 230.8^\circ, 309.2^\circ \text{ (both)}$	<p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1; A1 ✓</p> <p>M1A1 (7)</p> <p>[9]</p>
<p>Notes:</p>	<p>(a) N.B: AG; need to see at least one line of working after substituting $\cos^2 \theta$.</p> <p>(b) First M1: Using $5 \sin^2 \theta = 3$ to find value for $\sin \theta$ or θ</p> <p>[Allow such results as $\sin \theta = \frac{3}{5}$, $\sin \theta = \frac{\sqrt{3}}{5}$for M1]</p> <p>Second M1: Considering the – value for $\sin \theta$. (usually later)</p> <p>First A1: Given for awrt 50.8°. Not dependent on second M.</p> <p>Third M1: For $(180 - \text{candidate}' \text{ s } 50.8)^\circ$, need not see written down</p> <p>Final M1: Dependent on second M (but may be implied by answers)</p> <p>For $(180 + \text{candidate}' \text{ s } 50.8)^\circ$ or $(360 - \text{candidate}' \text{ s } 50.8)^\circ$ or equiv.</p> <p>Final A1: Requires both values. (no follow through)</p> <p>[Finds $\cos^2 \theta = k$ ($k = 2/5$) and so $\cos \theta = (\pm)...$M1, then mark equivalently]</p> <p>NB Candidates who only consider positive value for $\sin \theta$</p> <p>can score max of 4 marks: M1M0A1M1A1M0A0 – Very common.</p> <p>Candidates who score first M1 but have wrong $\sin \theta$ can score maximum</p> <p style="text-align: center;">M1M1A0M1A✓ M1A0</p> <p>SC Candidates who obtain one value from each set, e.g 50.8 and 309.2</p> <p style="text-align: center;">M1M1(bod)A1M0A0M1(bod)A0</p> <p>Extra values out of range – no penalty</p> <p>Any very tricky or “ outside scheme methods” , send to TL</p>	

5.	<p><u>Method 1</u> (Substituting $a = 3b$ into second equation at some stage)</p> <p>Using a law of logs correctly (anywhere) e.g. $\log_3 ab = 2$ M1</p> <p>Substitution of $3b$ for a (or $a/3$ for b) e.g. $\log_3 3b^2 = 2$ M1</p> <p>Using base correctly on correctly derived $\log_3 p = q$ e.g. $3b^2 = 3^2$ M1</p> <p>First correct value $b = \sqrt{3}$ (allow $3^{1/2}$) A1</p> <p>Correct method to find other value (dep. on at least first M mark) M1</p> <p>Second answer $a = 3b = 3\sqrt{3}$ or $\sqrt{27}$ A1</p> <p><u>Method 2</u> (Working with two equations in $\log_3 a$ and $\log_3 b$)</p> <p>“ Taking logs” of first equation and “ separating” $\log_3 a = \log_3 3 + \log_3 b$ M1 $(= 1 + \log_3 b)$</p> <p>Solving simultaneous equations to find $\log_3 a$ or $\log_3 b$ M1 $[\log_3 a = 1\frac{1}{2}, \log_3 b = \frac{1}{2}]$</p> <p>Using base correctly to find a or b M1</p> <p>Correct value for a or b $a = 3\sqrt{3}$ or $b = \sqrt{3}$ A1</p> <p>Correct method for second answer, dep. on first M; correct second answer M1;A1[6] [Ignore negative values]</p>	
Notes:	<p>Answers must be exact; decimal answers lose both A marks</p> <p>There are several variations on Method 1, depending on the stage at which $a = 3b$ is used, but they should all mark as in scheme.</p> <p>In this method, the first three method marks on Epen are for</p> <p>(i) First M1: correct use of log law,</p> <p>(ii) Second M1: substitution of $a = 3b$,</p> <p>(iii) Third M1: requires using base correctly on correctly derived $\log_3 p = q$</p> <p><u>Three examples of applying first 4 marks in Method 1:</u></p> <p>(i) $\log_3 3b + \log_3 b = 2$ gains second M1 $\log_3 3 + \log_3 b + \log_3 b = 2$ gains first M1 $(2\log_3 b = 1, \log_3 b = \frac{1}{2})$ no mark yet $b = 3^{1/2}$ gains third M1, and if correct A1</p> <p>(ii) $\log_3(ab) = 2$ gains first M1 $ab = 3^2$ gains third M1 $3b^2 = 3^2$ gains second M1</p> <p>(iii) $\log_3 3b^2 = 2$ has gained first 2 M marks $\Rightarrow 2\log_3 3b = 2$ or similar type of error $\Rightarrow \log_3 3b = 1 \Rightarrow 3b = 3$ does not gain third M1, as $\log_3 3b = 1$ not derived correctly</p>	

<p>6.</p>	 <p>(a) $BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700 \cos 15^\circ$ (= 63851.92...) $BC = 253$ awrt</p> <p>(b) $\frac{\sin B}{700} = \frac{\sin 15}{\text{candidate's } BC}$ $\sin B = \sin 15 \times 700 / 253_c = 0.716..$ and giving an obtuse B (134.2°) dep on 1st M $\theta = 180^\circ - \text{candidate's angle } B$ (Dep. on first M only, B can be acute) $\theta = 180 - 134.2 = (0)45.8$ (allow 46 or awrt 45.7, 45.8, 45.9) [46 needs to be from correct working]</p>	<p>M1 A1 A1 (3) M1 M1 M1 A1 (4) [7]</p>
<p>Notes:</p>	<p>(a) If use $\cos 15^\circ = \dots$, then A1 not scored until written as $BC^2 = \dots$ correctly</p> <p><i>Splitting into 2 triangles BAX and CAX, where X is foot of perp. from B to AC</i> Finding value for BX and CX and using Pythagoras M1 $BC^2 = (500 \sin 15^\circ)^2 + (700 - 500 \cos 15^\circ)^2$ A1 $BC = 253$ awrt A1</p> <p>(b) Several alternative methods: (Showing the M marks, 3rd M dep. on first M))</p> <p>(i) $\cos B = \frac{500^2 + \text{candidate's } BC^2 - 700^2}{2 \times 500 \times \text{candidate's } BC}$ or $700^2 = 500^2 + BC_c^2 - 2 \times 500 \times BC_c$ M1 Finding angle B M1 dep., then M1 as above</p> <p>(ii) 2 triangle approach, as defined in notes for (a) $\tan CBX = \frac{700 - \text{value for } AX}{\text{value for } BX}$ M1 Finding value for $\angle CBX$ ($\approx 59^\circ$) dep M1 $\theta = [180^\circ - (75^\circ + \text{candidate's } \angle CBX)]$ M1</p> <p>(iii) Using sine rule (or cos rule) to find C first: Correct use of sine or cos rule for C M1, Finding value for C M1 Either $B = 180^\circ - (15^\circ + \text{candidate's } C)$ or $\theta = (15^\circ + \text{candidate's } C)$ M1</p> <p>(iv) $700 \cos 15^\circ = 500 + BC \cos \theta$ M2 {first two Ms earned in this case} Solving for θ; $\theta = 45.8$ (allow 46 or 5.7, 45.8, 45.9) M1;A1</p> <p>Note: S.C. In main scheme, if θ used in place of B, third M gained immediately; Other two marks likely to be earned, too, for correct value of θ stated.</p>	

7	<p>(a) Either solving $0 = x(6 - x)$ and showing $x = 6$ (and $x = 0$) or showing $(6,0)$ (and $x = 0$) satisfies $y = 6x - x^2$ [allow for showing $x = 6$]</p> <p>(b) Solving $2x = 6x - x^2$ ($x^2 = 4x$) to $x = \dots$ $x = 4$ (and $x = 0$) Conclusion: when $x = 4$, $y = 8$ and when $x = 0$, $y = 0$,</p> <p>(c) (Area =) $\int_{(0)}^{(4)} (6x - x^2) dx$ Limits not required Correct integration $3x^2 - \frac{x^3}{3} (+c)$ Correct use of correct limits on their result above (see notes on limits) $[\frac{3x^2 - x^3}{3}]^4 - [\frac{3x^2 - x^3}{3}]_0$ with limits substituted $[= 48 - 21\frac{1}{3} = 26\frac{2}{3}]$ Area of triangle = $2 \times 8 = 16$ (Can be awarded even if no M scored, i.e. B1) Shaded area = \pm (area under curve - area of triangle) applied correctly $(= 26\frac{2}{3} - 16) = 10\frac{2}{3}$ (awrt 10.7)</p>	<p>B1 (1)</p> <p>M1 A1 A1 (3)</p> <p>M1 A1 M1 A1 M1 A1 (6)[10]</p>
Notes	<p>(b) In scheme first A1: need only give $x = 4$ If <i>verifying approach</i> used: Verifying $(4,8)$ satisfies both the line and the curve M1(attempt at both), Both shown successfully A1 For final A1, $(0,0)$ needs to be mentioned; accept "clear from diagram"</p> <p>(c) Alternative Using Area = $\pm \int_{(0)}^{(4)} \{(6x - x^2); -2x\} dx$ approach</p> <p>(i) If candidate integrates separately can be marked as main scheme If combine to work with = $\pm \int_{(0)}^{(4)} (4x - x^2) dx$, first M mark and third M mark $= (\pm) [2x^2 - \frac{x^3}{3} (+c)]$ A1, Correct use of correct limits on their result second M1, Totally correct, unsimplified \pm expression (may be implied by correct ans.) A1 $10\frac{2}{3}$ A1 [Allow this if, having given $-10\frac{2}{3}$, they correct it]</p> <p>M1 <i>for correct use of correct limits</i>: Must substitute correct limits for their strategy into a changed expression and subtract, either way round, e.g. $\pm \{ []^4 - []_0 \}$ If a long method is used, e.g, finding three areas, this mark only gained for correct strategy and all limits need to be correct for this strategy. Final M1: limits for area under curve and triangle must be the same.</p> <p>S.C.(1) $\int_0^6 (6x - x^2) dx - \int_0^6 2x dx = \left[3x^2 - \frac{x^3}{3} \right]_0^6 - [x^2]_0^6 = \dots$ award M1A1 MO(limits)AO(triangle)M1(bod)A0</p> <p>(2) If, having found \pm correct answer, thinks this is not complete strategy and does more, do not award final 2 A marks</p> <p>Use of trapezium rule: M0A0MA0possibleA1for triangle M1(if correct application of trap. rule from $x = 0$ to $x = 4$) A0</p>	

8	<p>(a) $(x-6)^2 + (y-4)^2 = 3^2$</p> <p>(b) Complete method for MP: $= \sqrt{(12-6)^2 + (6-4)^2}$ $= \sqrt{40}$ or awrt 6.325</p> <p>[These first two marks can be scored if seen as part of solution for (c)]</p> <p>Complete method for $\cos \theta$, $\sin \theta$ or $\tan \theta$ e.g. $\cos \theta = \frac{MT}{MP} = \frac{3}{\text{candidate's } \sqrt{40}}$ ($= 0.4743$) ($\theta = 61.6835^\circ$) [If $TP = 6$ is used, then M0] $\theta = 1.0766$ rad AG</p> <p>(c) Complete method for area TMP; e.g. $= \frac{1}{2} \times 3 \times \sqrt{40} \sin \theta$ $= \frac{3}{2} \sqrt{31}$ ($= 8.3516..$) allow awrt 8.35</p> <p>Area (sector) $MTQ = 0.5 \times 3^2 \times 1.0766$ ($= 4.8446..$)</p> <p>Area $TPQ = \text{candidate's } (8.3516.. - 4.8446..)$ $= 3.507$ awrt [Note: 3.51 is A0]</p>	<p>B1; B1 (2)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (4)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1 (5)</p> <p>[11]</p>
Notes	<p>(a) Allow 9 for 3^2.</p> <p>(b) First M1 can be implied by $\sqrt{40}$ or $\sqrt{31}$</p> <p>For second M1: May find $TP = \sqrt{(\sqrt{40})^2 - 3^2} = \sqrt{31}$, then either $\sin \theta = \frac{TP}{MP} = \frac{\sqrt{31}}{\sqrt{40}}$ ($= 0.8803..$) or $\tan \theta = \frac{\sqrt{31}}{3}$ ($1.8859..$) or cos rule</p> <p>NB. Answer is given, but allow final A1 if all previous work is correct.</p> <p>(c) First M1: (alternative) $\frac{1}{2} \times 3 \times \sqrt{40 - 9}$</p> <p>Second M1: allow even if candidate's value of θ used. (Despite being given !)</p>	

9	<p>(a) (Total area) = $3xy + 2x^2$ (Vol:) $x^2y = 100$ $(y = \frac{100}{x^2}, xy = \frac{100}{x})$</p> <p>(b) Deriving expression for area in terms of x only (Substitution, or clear use of, y or xy into expression for area) (Area =) $\frac{300}{x} + 2x^2$ AG</p> <p>(c) $\frac{dA}{dx} = -\frac{300}{x^2} + 4x$ Setting $\frac{dA}{dx} = 0$ and finding a value for correct power of x, for cand. M1 [$x^3 = 75$] $x = 4.2172$ awrt 4.22 (allow exact $\sqrt[3]{75}$) $\frac{d^2A}{dx^2} = \frac{600}{x^3} + 4 = \text{positive}, > 0;$ therefore minimum</p> <p>(d) Substituting found value of x into (a) (Or finding y for found x and substituting both in $3xy + 2x^2$) [$y = \frac{100}{4.2172^2} = 5.6228$] Area = 106.707 awrt 107</p>	<p>B1 B1 M1 A1 cso (4) M1A1 A1 (4) M1;A1 (2) M1 A1 (2) [12]</p>
Notes	<p>(a) First B1: Earned for correct unsimplified expression, isw.</p> <p>(b) First M1: At least one power of x decreased by 1, and no “c” term.</p> <p>(c) For M1: Find $\frac{d^2A}{dx^2}$ and explicitly consider its sign, state > 0 or “positive” A1: Candidate’s $\frac{d^2A}{dx^2}$ must be correct for their $\frac{dA}{dx}$, sign must be + ve and conclusion “so minimum”, (allow QED, \checkmark). (may be wrong x, or even no value of x found) <u>Alternative:</u> M1: Find value of $\frac{dA}{dx}$ on either side of “$x = \sqrt[3]{75}$” and consider sign A1: Indicate sign change of negative to positive for $\frac{dA}{dx}$, and conclude minimum. OR M1: Consider values of A on either side of “$x = \sqrt[3]{75}$” and compare with “107” A1: Both values greater than “$x = 107$” and conclude minimum. Allow marks for (c) and (d) where seen; even if part labelling confused. Throughout, allow confused notation, such as dy/dx for dA/dx.</p>	

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