Rewarding Learning

ADVANCED
General Certificate of Education
January 2014

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

AMF21

MONDAY 27 JANUARY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(i)** Simplify the expression

$$(\sqrt{r} + \sqrt{r-1})(\sqrt{r} - \sqrt{r-1}) \qquad [1]$$

(ii) Hence show that $$\sum_{r=1}^{n} \frac{1}{\sqrt{r} + \sqrt{r-1}} = \sqrt{n} \qquad [3]$$

2 Find, in radians, the general solution of the equation

$$3 \tan^2 \theta = 2 \sin \theta \qquad [9]$$

3 **(a)** A root of the equation

$$z^4 - 6z^3 + 23z^2 - 34z + 26 = 0$$

is $1 + \mathrm{i}$

(i) State with a reason one other complex root. [2]

(ii) Find the remaining roots. [5]

(b) Given that n is a positive integer, show that

$$(1 + \mathrm{i})^{4n} - (1 - \mathrm{i})^{4n} = 0 \qquad [5]$$

4 Let $y = \tan x$

(i) Show that $\dfrac{dy}{dx} = 1 + y^2$ and $\dfrac{d^2y}{dx^2} = 2y(1 + y^2)$ [3]

(ii) Find, in terms of y, an expression for $\dfrac{d^3y}{dx^3}$ [2]

(iii) Hence or otherwise find the Maclaurin series for $\tan x$ up to and including the term in x^3 [4]

(iv) Using this approximation for $\tan x$ and other small angle approximations, find

$$\lim_{x \to 0} \left(\frac{x \sin^2 x}{\tan x - x \cos x} \right)$$ [3]

5 Use mathematical induction to prove that if $n \geqslant 1$ and $\sin x \neq 0$

$$\cos x + \cos 3x + \cos 5x + \ldots + \cos(2n - 1)x \equiv \frac{\sin 2nx}{2 \sin x}$$ [9]

6 The extension x centimetres of a spiral spring at time t seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + n^2 x = \cos \omega t$$

where n and ω are constants and $\omega \neq n$

Given that when $t = 0$, $x = 0$ and $\dfrac{dx}{dt} = V$, express x as a function of t. [12]

7 **(i)** Find the eccentricity of the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$ [2]

(ii) State the coordinates of the foci and the equations of the directrices. [2]

(iii) Show that an equation for the tangent to this ellipse at the point $P(3\cos\theta, 2\sin\theta)$ is

$$\frac{x\cos\theta}{3} + \frac{y\sin\theta}{2} = 1$$ [4]

A line perpendicular to the tangent at P is drawn from the origin and intersects the tangent at Q.

(iv) Find, in terms of θ, the coordinates of Q. [5]

(v) Verify that as θ varies Q lies on the curve

$$(x^2 + y^2)^2 = 9x^2 + 4y^2$$ [4]

THIS IS THE END OF THE QUESTION PAPER