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General Certificate of Education (A-level)
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Mathematics MFP4

(Specification 6360)

Further Pure 4

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1	Attempt at $\begin{vmatrix}3 & 2 & p\\ 7 & -1 & 6\\ 2 & 1 & 3\end{vmatrix} = 0$	M1		
	Solving a linear equation in p	M1		
	$p = 5$	A1	3	
	ALT Lin.dep. iff $\exists$ constants a and b s.t. $a\mathbf{u} + b\mathbf{v} = \mathbf{w}$			
	Solving simultaneously $3a + 7b = 2$ and $2a - b = 1$ M1			(from **i** and **j** components)
	$a = \frac{9}{17}$, $b = \frac{1}{17}$ A1			
	Substituting back into **k** component ($ap + 6b = 3$) to find "their" p correctly A1F			
	Total		**3**	
2(a)	Choice of $\begin{pmatrix}4\\ 7\\ -4\end{pmatrix}$ as direction vector	B1		
	$\sqrt{4^2+7^2+4^2}$ or $\sqrt{3^2+2^2+6^2}$	M1		Either attempted
	$\frac{4}{9}, \frac{7}{9}, -\frac{4}{9}$ or $\frac{3}{7}, -\frac{2}{7}, \frac{6}{7}$	A1	3	ft their chosen direction vector
(b)	Direction cosines are the **cosines** of the angles between the line and the coordinate axes	B1	1	
	Total		**4**	
3(a)	eg $\begin{vmatrix}yz & x(y+z) & xy\\ x & y+z & z\\ x^2 & z^2-y^2 & z^2\end{vmatrix}$	M1		$C_2' = C_2 + C_3$
	$= (y+z)\begin{vmatrix}yz & x & xy\\ x & 1 & z\\ x^2 & z-y & z^2\end{vmatrix}$	A1	2	
(b)	eg $(y+z)\begin{vmatrix}y(z-x) & x & xy\\ x-z & 1 & z\\ x^2-z^2 & z-y & z^2\end{vmatrix}$ $C_1' = C_1 - C_3$	M1		Attempt at a second linear factor
	$= (x-z)(y+z)\begin{vmatrix}-y & x & xy\\ 1 & 1 & z\\ x+z & z-y & z^2\end{vmatrix}$	A1		
	$= (x-z)(y+z)\begin{vmatrix}-(x+y) & x & xy\\ 0 & 1 & z\\ x+y & z-y & z^2\end{vmatrix}$	M1		$C_1' = C_1 - C_2$ Complete attempt at remaining factors (M0 if they just expand and can do nothing with it)
	$= (x-z)(y+z)(x+y)\left(xz - xy - yz\right)$	A1	4	
	Total		**6**	

Q	Solution	Marks	Total	Comments
4(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -4 & 7 \\ 2 & -2 & 3 \end{vmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$	M1		Attempt at the vector product of the 2 d.v.s **OR** two scalar products = 0 and some manipulation attempt
		A1	2	
(b)(i)	$\overrightarrow{AB} = \begin{bmatrix} 2\beta+7 \\ 19-2\beta \\ 3\beta-2 \end{bmatrix} - \begin{bmatrix} 3\alpha+7 \\ -25-4\alpha \\ 7\alpha+9 \end{bmatrix}$			
	$= \begin{bmatrix} 2\beta-3\alpha \\ 44-2\beta+4\alpha \\ -11+3\beta-7\alpha \end{bmatrix}$	M1		Good attempt
	$\overrightarrow{AB} = \lambda\mathbf{n} = \begin{bmatrix} 2\lambda \\ 5\lambda \\ 2\lambda \end{bmatrix}$	M1		With their $\overrightarrow{AB}$ (involving α and β) and their **n**
	Legitimately getting given system of equations: $3\alpha - 2\beta + 2\lambda = 0$ $4\alpha - 2\beta - 5\lambda = -44$ $7\alpha - 3\beta + 2\lambda = -11$	A1	3	
(ii)	Solving this 3 × 3 system in α, β and λ (possibly just λ given or both α, β)	M1		
	For $\alpha = -2$, $\beta = 3$ ($\lambda = 6$)	A1		
	$A = (1, -17, -5)$ and $B = (13, 13, 7)$	A1	3	Give one A1 for a correct pair (α, A) or (β, B) **SC**: allow all 3 marks ft for misreads of the signs (44 and/or 11 only)
(iii)	Shortest distance $= \sqrt{12^2 + 30^2 + 12^2}$ or $\lvert \lambda \rvert \lvert \mathbf{n} \rvert = 6\sqrt{2^2 + 5^2 + 2^2}$	M1		Allow also for the shortest distance formula $\lvert(\mathbf{b} - \mathbf{a}) \cdot \hat{\mathbf{n}}\rvert$
	$6\sqrt{33}$, $\dfrac{198}{\sqrt{33}}$, $\sqrt{1188}$ or AWRT 34.5	A1	2	CAO
	Total		**10**	

Q	Solution	Marks	Total	Comments
5(a)(i)	Char. eqn. is $\lambda^2 - 2\lambda + 1 = 0$	M1		Written down or attempted via determinant
	$\lambda = 1$ (twice)	A1		
	Substituting $\lambda = 1 \Rightarrow -12x + 9y = 0$ and/or $-16x + 12y = 0$	M1		
	Eigenvector(s) $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$	A1	4	Any non-zero multiple will do
(ii)	$m = \frac{4}{3}$	B1		ft
	LOIPs since $\lambda = 1$ (or full description)	B1	2	
(iii)	$\begin{bmatrix} -11 & 9 \\ -16 & 13 \end{bmatrix}\begin{bmatrix} x \\ \frac{4}{3}x + c \end{bmatrix}$	M1		m must be their numerical value
	$= \begin{bmatrix} -11x + 9\left(\frac{4}{3}x + c\right) \\ -16x + 13\left(\frac{4}{3}x + c\right) \end{bmatrix}$ or $\begin{bmatrix} x + 9c \\ \frac{4}{3}x + 13c \end{bmatrix}$	A1		ft
	For showing $y' = \frac{4}{3}x' + c$	B1	3	Impossible without correct prior working
(b)	... parallel to $y = \frac{4}{3}x$"	B1		ft their m
	For mapping any one point to a correct image point, eg (1, 0) to (–11, –16), (0, 1) to (9, 13) or (1, 1) to (–2, –3)	B1	2	If they cite multiple points, including incorrect ones, then B0
(c)	Shear parallel to $y = \frac{4}{3}x$	M1		MUST be the same LOIPs as in (b)
	For mapping any one point to a correct image point	A1	2	ft incorrect ones ONLY if they are the reverse of those cited in (b)
	Total		**13**	

Q	Solution	Marks	Total	Comments
6(a)	For attempt at/getting normal d.v. $\begin{bmatrix}1\\5\\7\end{bmatrix}$	M1 A1		
	For getting a point on the line eg (0, –8, –11)	M1 A1		
	For line equation: ($\mathbf{r}$ =) (their point) + λ (their d.v.)	B1	5	Condone lack of r or **r** here
	ALT eg Adding Π_1 and Π_2 M1 $\Rightarrow$ $5x - y = 8$ A1 Setting (eg) $x=\lambda$ and getting y, z in terms of λ: $y = 5\lambda - 8$, $z = 7\lambda - 11$ M1 Turning this into a vector equation of the given form M1 $(\mathbf{r} =) \begin{bmatrix}0\\-8\\-11\end{bmatrix} + \lambda\begin{bmatrix}1\\5\\7\end{bmatrix}$ A1			
(b)(i)	Substituting their x, y, z in terms of λ into Π_3's equation ($12x - y - z = 40$)	M1		
	For correct statement with no λ's in: (19 = 40)	A1		ft on incorrect values from their "point"
	Correct conclusion, from valid working, that the system is inconsistent	B1	3	or "consistent" if it genuinely yields 40 = 40
(ii)	The three planes form a (triangular) prism – allow clear diagram	B1	1	ft "sheaf" from a "consistent" conclusion
(c)(i)	e.g. $\Pi_2 + \Pi_3 \Rightarrow 15x - 3y = 45$ Setting $x = 0$ (eg)	M1 M1		
	Any correct point, eg (0, –15, –25), (3, 0, –4), $\left(\frac{25}{7}, \frac{20}{7}, 0\right)$, (4, 5, 3) etc	A1	3	
(ii)	$\mathbf{r} = \begin{bmatrix}3\\0\\-4\end{bmatrix} + \lambda\begin{bmatrix}1\\5\\7\end{bmatrix}$	B1	1	ft their common point and d.v. from (a) Penalise missing r or **r** here
			13	

Q	Solution	Marks	Total	Comments
7(a)(i)	$\mathbf{A}^{\mathrm{T}} = \begin{bmatrix} k & 2 & 1 \\ 1 & k & 2 \\ 2 & 1 & k \end{bmatrix}$	B1		
	Good multiplication attempt at $\mathbf{A}\,\mathbf{A}^{\mathrm{T}}$	M1		
	$= \begin{bmatrix} k^2+5 & 3k+2 & 3k+2 \\ 3k+2 & k^2+5 & 3k+2 \\ 3k+2 & 3k+2 & k^2+5 \end{bmatrix}$	A1 A1		Main diagonal correct All others correct
	$k = -\frac{2}{3}$ $\quad m = 5\frac{4}{9}$ or $\frac{49}{9}$	A1A1	6	
				If they multiply $\mathbf{A}^{\mathrm{T}}\,\mathbf{A}$ instead, they can score B1 M1 A0 A0 A1 A1
(ii)	$\begin{bmatrix} -\frac{2}{3} & 1 & 2 \\ 2 & -\frac{2}{3} & 1 \\ 1 & 2 & -\frac{2}{3} \end{bmatrix}^{-1} = \frac{9}{49}\begin{bmatrix} -\frac{2}{3} & 2 & 1 \\ 1 & -\frac{2}{3} & 2 \\ 2 & 1 & -\frac{2}{3} \end{bmatrix}$	B1	1	ft $\dfrac{1}{\text{their } m}$ and their k
	Accept $\frac{9}{49}\begin{bmatrix} k & 2 & 1 \\ 1 & k & 2 \\ 2 & 1 & k \end{bmatrix}$ since the value of k is now known, but not just $\frac{9}{49}\mathbf{A}^{\mathrm{T}}$			
	Decimal version (correct to at least 3sf) is also ok: $\begin{bmatrix} -0.122 & 0.367 & 0.184 \\ 0.184 & -0.122 & 0.367 \\ 0.367 & 0.184 & -0.122 \end{bmatrix}$			
(b)(i)	det $\mathbf{A} = k^3 - 6k + 9$	M1 A1	2	Good attempt (cubic)
(ii)	det $\mathbf{A} = (k+3)(k^2 - 3k + 3)$ $k = -3$ $\Delta = 9 - 12 < 0 \Rightarrow$ no further real roots	M1 A1 B1	3	Factorisation attempt **Special Cases** $k = -3$ but no working B1 For all 3 roots, -3 and $\dfrac{3 \pm \mathrm{i}\sqrt{3}}{2}$ given with no supporting working B3
(iii)	Replacing k by $(k-7)$ $k = 4$ (however obtained)	M1 A1	2	NOT just in the determinant form (ie starting again) ft their previous $k + 7$
			14	

Q	Solution	Marks	Total	Comments
8(a)	$\begin{bmatrix}7\\4\\6\end{bmatrix}\cdot\begin{bmatrix}2\\1\\3\end{bmatrix} = 14 + 4 + 18 = 36 \Rightarrow Q \text{ in } \Pi$	B1	1	
(b)	$\begin{bmatrix}-7\\5\\3\end{bmatrix}\cdot\begin{bmatrix}2\\1\\3\end{bmatrix} = -14 + 5 + 9 = 0$	B1		Shown
	Explanation that l is perpendicular to Π's normal $\Rightarrow$ l is parallel to Π	B1	2	
	ALT $\begin{bmatrix}20-7\mu\\5\mu-8\\3\mu+1\end{bmatrix}\cdot\begin{bmatrix}2\\1\\3\end{bmatrix} = 35 \ \ (+\ 0\mu)$ B1 Explanation that, since this $\neq 36$ (and constant), l does not intersect Π and must therefore be parallel to it B1			
(c)	**Mark according to whichever scheme gives the greatest credit**			
	MARK SCHEME I			
	For $\mathbf{p} = \mathbf{q} + \lambda\,\mathbf{n} = \begin{bmatrix}7+2\lambda\\4+\lambda\\6+3\lambda\end{bmatrix}$ for some λ	B1		
	For $PQ = \lvert\lambda\rvert\sqrt{14}$ or equivalent	B1		
	$\overrightarrow{PR} = \mathbf{r} - \mathbf{p} = \begin{bmatrix}20-7\mu\\5\mu-8\\3\mu+1\end{bmatrix} - \begin{bmatrix}7+2\lambda\\4+\lambda\\6+3\lambda\end{bmatrix}$	M1		
	$= \begin{bmatrix}-7\mu-2\lambda+13\\5\mu-\lambda-12\\3\mu-3\lambda-5\end{bmatrix}$	A1		
	Setting $\overrightarrow{PR}\cdot\begin{bmatrix}-7\\5\\3\end{bmatrix} = 0$	M1		
	Solving linear equation for λ	M1		$\Rightarrow 83\mu - 166 = 0$ ultimately
	$\mu = 2$ and/or $\overrightarrow{PR} = \begin{bmatrix}-(2\lambda+1)\\-(\lambda+2)\\1-3\lambda\end{bmatrix}$	A1		
	$PR^2 = PQ^2$	M1		$(1+2\lambda)^2 + (2+\lambda)^2 + (1-3\lambda)^2 = 14\lambda^2$
	$(\lambda = -3)$ $\quad P = (1, 1, -3)$	A1	9	

Q	Solution	Marks	Total	Comments
8(c) cont	**MARK SCHEME II**			
	For $\mathbf{p} = \mathbf{q} + \lambda\,\mathbf{n} = \begin{bmatrix} 7+2\lambda \\ 4+\lambda \\ 6+3\lambda \end{bmatrix}$ for some λ	B1		
	For $PQ^2 = 14\lambda^2$	B1		
	$\overrightarrow{PR} = \mathbf{r} - \mathbf{p} = \begin{bmatrix} 20-7\mu \\ 5\mu-8 \\ 3\mu+1 \end{bmatrix} - \begin{bmatrix} 7+2\lambda \\ 4+\lambda \\ 6+3\lambda \end{bmatrix}$	M1		
	$= \begin{bmatrix} -7\mu-2\lambda+13 \\ 5\mu-\lambda-12 \\ 3\mu-3\lambda-5 \end{bmatrix}$	A1		
	$PR^2 = (-13 + 7\mu + 2\lambda)^2$ $+ (12 - 5\mu + \lambda)^2 + (5 - 3\mu + 3\lambda)$	M1		
	Setting $PR^2 = PQ^2$ $\Rightarrow\ 83\mu^2 - 332\mu + 338 + 2\lambda = 0$	M1 A1		Correct quadratic in μ
	Considering discriminant = 0 or $83(\mu - 2)^2 = -2(\lambda + 3)$	M1		
	$\mu = 2$, $\lambda = -3$ and $P = (1, 1, -3)$	A1	(9)	
	MARK SCHEME III			
	For $\mathbf{p} = \mathbf{q} + \lambda\,\mathbf{n} = \begin{bmatrix} 7+2\lambda \\ 4+\lambda \\ 6+3\lambda \end{bmatrix}$ for some λ	B1		
	For $PQ^2 = 14\lambda^2$	B1		
	For $\mathbf{r} = \begin{bmatrix} 20-7\mu \\ 5\mu-8 \\ 3\mu+1 \end{bmatrix}$	B1		
	For $QR^2 =$ $(13 - 7\mu)^2 + (5\mu - 12)^2 + (3\mu - 5)^2$ $= 83\mu^2 - 332\mu + 338 = 83(\mu - 2)^2 + 6$	M1		
	For R closest to Q when $\mu = 2$, $R = (6, 2, 7)$	M1 A1		
	Then $\overrightarrow{RP} = \begin{bmatrix} 1+2\lambda \\ 2+\lambda \\ 3\lambda-1 \end{bmatrix}$	B1		ft
	Setting $PR^2 = PQ^2$ $\Rightarrow\ (1 + 2\lambda)^2 + (2 + \lambda)^2 + (3\lambda - 1)^2 = 14\lambda^2$	M1		
	$(\lambda = -3)$ $\quad P = (1, 1, -3)$	A1	(9)	
	Total		**12**	
	TOTAL		**75**	