Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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TOTAL	

General Certificate of Education
Advanced Level Examination
June 2012

Mathematics MM05

Unit Mechanics 5

Monday 25 June 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A simple pendulum has a length of 0.5 metres.

(a) Find the period of motion of this pendulum. *(2 marks)*

(b) Find the frequency of motion of this pendulum. *(1 mark)*

QUESTION PART REFERENCE

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2 A particle P, of mass 0.4 kg, is attached to one end of a light elastic string, and the other end of the string is attached to a fixed point A.

(a) The particle P hangs in equilibrium at a point E, vertically below A, where the extension of the string is 0.2 metres. Calculate the stiffness of the string. *(3 marks)*

(b) The particle P is pulled vertically downwards from the point E by a distance of 0.1 metres, and released from rest. The displacement of P from E at time t seconds after being released is x metres.

(i) Given that the string does not become slack during the subsequent motion, show that

$$\ddot{x} = hx$$

where h is a constant to be determined. *(4 marks)*

(ii) Hence deduce that the motion of P is simple harmonic. *(1 mark)*

(iii) Show that the period of this motion is 0.898 seconds, correct to three significant figures. *(2 marks)*

(iv) Calculate the maximum speed of P during its motion. *(2 marks)*

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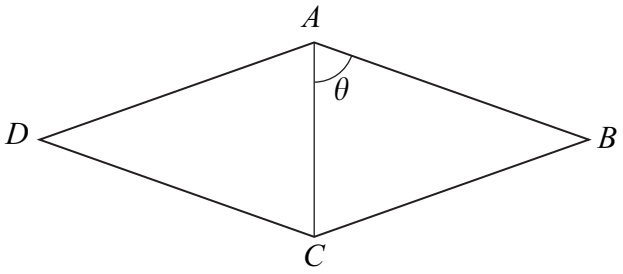
3 Four identical uniform rods, each of length $2a$ and weight W, are smoothly hinged together to form a rhombus $ABCD$. The vertex A of the rhombus is smoothly pinned to a fixed support. A light elastic string connects the vertices A and C of the rhombus. The string is of natural length a and modulus of elasticity $2W$. The rhombus hangs in equilibrium, and the angle between the rod AB and the string is θ, as shown in the diagram, where $0 < \theta < \frac{\pi}{2}$.



Gravitational potential energy is taken to be zero at the level of A.

(a) Show that the potential energy, V, of the system is given by

$$V = -8Wa\cos\theta + Wa(4\cos\theta - 1)^2$$ *(6 marks)*

(b) (i) Find the only value for θ for which the system is in equilibrium. *(5 marks)*

(ii) Determine whether the system is in stable or unstable equilibrium for this value of θ. *(3 marks)*

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4 A particle P, of mass m kg, moves in a straight horizontal line. At time t seconds, the displacement of P from a fixed point O on the line is x metres, and P is moving with velocity $\dot{x}\,\mathrm{m\,s^{-1}}$. Throughout the motion, two horizontal forces act on P: a force of magnitude $4mn^2|x|$ newtons directed towards O, and a resistance force of magnitude $2mk|\dot{x}|$ newtons, where n and k are positive constants.

(a) Show that $\ddot{x} + 2k\dot{x} + 4n^2x = 0$. *(3 marks)*

(b) In one case, $k = n$. When $t = 0$, $x = a$ and $\dot{x} = 0$.

(i) Show that $x = \mathrm{e}^{-nt}\left(a\cos\sqrt{3}nt + \left(\frac{\sqrt{3}a}{3}\right)\sin\sqrt{3}nt\right)$. *(7 marks)*

(ii) Show that P passes through O when $\tan\sqrt{3}nt = -\sqrt{3}$. *(3 marks)*

(c) In a different case, $k = 2n$.

(i) Find a general solution for x at time t seconds. *(3 marks)*

(ii) Hence state the type of damping which occurs. *(1 mark)*

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5 A particle P, of mass m, is attached to one end of a light elastic string. The other end of the string is attached to a fixed point O on a smooth horizontal table. The string is of natural length a and modulus of elasticity $2mg$.

The string is extended to a length $3a$, and P is projected on the table at right angles to the string with speed U, as shown in the diagram.

During the subsequent motion, the polar coordinates of P with respect to O are (r, θ).

(a) (i) Explain why the transverse component of the acceleration of P is zero. *(1 mark)*

(ii) Find, in terms of a and U, an expression for $r^2\dot{\theta}$. *(2 marks)*

(b) During the motion, the maximum value of r is $4a$.

(i) Show that the speed of P in the position when r is at its maximum is $\frac{3U}{4}$. *(3 marks)*

(ii) Find an expression, in terms of a, g, m and U, for the total energy of the system at the moment of projection. *(3 marks)*

(iii) Hence find, in terms of a and g, the value of U. *(4 marks)*

(iv) When r is at its maximum, find the magnitude and direction of the acceleration of P. *(3 marks)*

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6 A small raindrop is falling through a large stationary cloud. During its motion through the cloud, the raindrop acquires moisture from the cloud. Air resistance on the raindrop is negligible.

During this motion, the mass of the raindrop at time t seconds is m kg, and the rate of increase of m is $km\,\text{kg}\,\text{s}^{-1}$, where k is a positive constant.

(a) When $t = 0$, the value of m is m_0. At time T, the raindrop has doubled its mass to $2m_0$, and at this time the raindrop leaves the cloud.

Show that $T = \frac{1}{k}\ln 2$. *(4 marks)*

(b) The velocity of the raindrop at time t seconds whilst falling through the cloud is $v\,\text{m}\,\text{s}^{-1}$.

By considering the change of momentum of the raindrop during a small time interval δt, show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g - kv$$ *(4 marks)*

(c) The velocity v is given by the expression

$$v = \frac{g}{k} - \left(\frac{g}{k} - U\right)\mathrm{e}^{-kt}$$

where $U\,\text{m}\,\text{s}^{-1}$ is the velocity of the raindrop when $t = 0$.

Verify that this expression satisfies the differential equation in part **(b)**. *(3 marks)*

(d) Given that $k = 0.7$ and $U = 2$, find the value of v when the raindrop leaves the cloud. *(2 marks)*

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END OF QUESTIONS

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