RADLEY COLLEGE
Entrance Scholarships

MATHEMATICS II

Thursday 7th March 2002

Time allowed 2 hours

You may try the questions in any order and you are not expected to complete them all.

Show all working.

1. I am considering buying a mobile phone, and have the choice of two suppliers - Pink and Green. Pink will charge me £10 per month plus 10p for every minute I use. Green makes no monthly charge, but calls cost 25p per minute.

(a) In one month I am on the phone for fifty minutes. How much will I pay under each scheme?

(b) Use algebra to find how long I have to be on the phone in one month for the costs from each supplier to be the same.

I decide to go with Pink, and they give me a discount of 15% on my first month's bill.

(c) Given I pay £19.72 for my first month with Pink, for how many minutes was I on the phone?

2. In a box is a mix of 5p and 20p coins. There are twenty eight coins in all, and the total value of the coins is £3.80.

Use Simultaneous Equations to calculate how many of each coin are in the box.

3. A circular rug of radius 3 m has a frill stitched all the way around its circumference.

(a) Calculate the length of the frill, leaving your answer as a multiple of π.

The frill is removed from the 3 m rug and shortened so as to fit a circular rug of radius 2 m.

(b) Leaving your answer as a multiple of π, calculate how much of the original frill is wasted.

(c) Prove that for any two circular rugs whose radii differ by 1 m, the difference in circumferences is always the same.

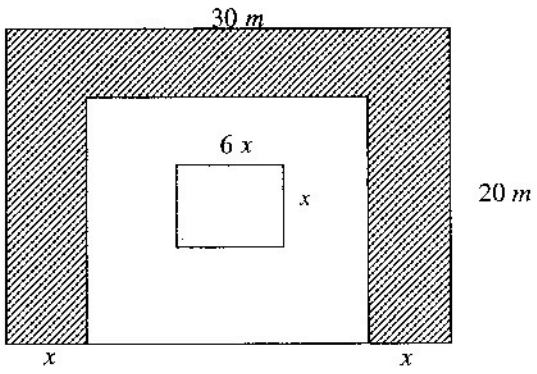
4. A train averages 60 km/h for the first 30 km of its journey, and then averages 100 km/h for the second part of its journey. The average speed for the whole journey is 90 km/h. Letting x km denote the length of the journey,

(a) write down an expression, in terms of x, for the total time of the whole journey,

(b) write down an expression, in terms of x, for the time of the second part of the journey,

(c) form an equation for x, and solve it.

5.



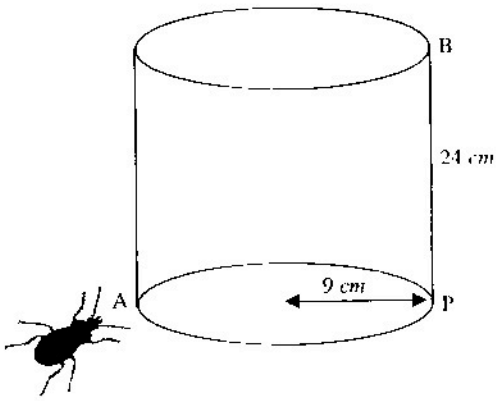
The diagram, not to scale, shows the plan of a rectangular garden, 30 metres long by 20 metres wide. Around three sides is a gravelled path of width x metres and in the middle is a rectangular pond, $(6x)$ metres long by x metres wide. The rest of the garden is grassed.

(a) Given that the area of the grass is $400\ m^2$, deduce the equation

$$2x^2 + 35x - 100 = 0$$

(b) Hence find the value of x.

6.



This year the Radley College Scholarship Beetle is tackling a Stilton. The Stilton is a cylinder of radius 9 cm and height 24 cm. The beetle is at A and wishes to get to B. P is the point which is diametrically opposite to A, and B is vertically above P.

Find how far it goes if it

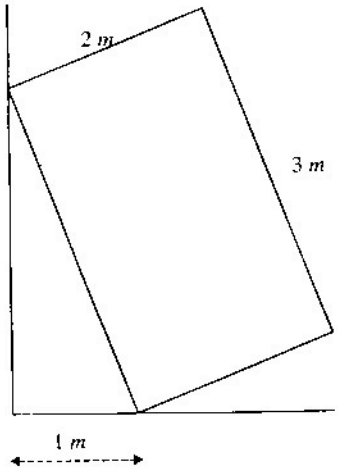
(a) burrows in a straight line from A to P, and then crawls straight up the edge from P to B,

(b) crawls around the edge of the circular base from A to P, and then crawls straight up the edge from P to B,

(c) burrows in a straight line from A to B,

(d) crawls over the surface, by the shortest possible such route, from A to B.

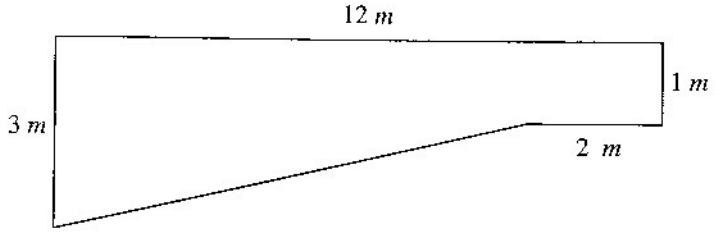
7.



A wardrobe of rectangular cross-section, 3 metres by 2 metres, is resting against a vertical wall. The bottom corner of the wardrobe is 1 metre from the wall, as shown in the diagram above.

Calculate the height of the highest point of the wardrobe above the ground.

8. A swimming pool is 12 m long by 6 m wide. The shallow end is 1 m deep and the deep end is 3 m deep. The pool starts to get deeper 2 m from the shallow end. The diagram shows the cross-section of the pool.



(a) Given that 1000 litres = 1 m^3, calculate, in litres, the volume of the pool.

In order to fill the pool a pipe of circular cross-section, radius 3 cm, is used. Water flows through the pipe at a rate of 40 cm/s.

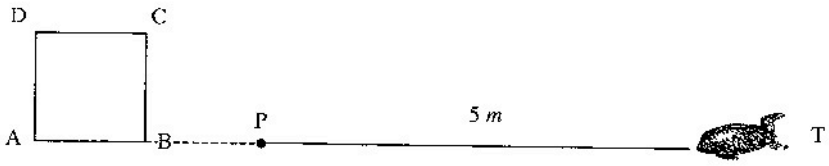
(b) Calculate how many litres of water enter the pool each hour.

(c) How long, to the nearest minute, will it take to fill the pool?

20 hours after filling begins the water supply ceases.

(d) How far below the top does the water reach at this time?

9.



The diagram shows a square box ABCD, of side 1 m. The Radley College Scholarship Tortoise, T, is attached by a string of length 5 m to a peg P. Initially ABPT lie in a straight line, and BP = 1 m.

(a) The Radley College Scholarship Tortoise sets off in a clockwise direction keeping the string taut. Find how far he walks before bumping into the box at B.

(b) If instead the tortoise had set off in anti-clockwise direction, how far would he walk, and where would he bump into the box?

10. Solve the simultaneous equations

$2x + 3y = 35$, $3x - y = 3$

Use your answers to solve each of the following pairs of simultaneous equations.

(i) $2x + 3y = 350$, $3x - y = 30$

(ii) $2x + 3y = -35$, $3x - y = -3$

(iii) $2x^2 + 3y^2 = 35$, $3x^2 - y^2 = 3$

(iv) $\frac{2}{x} + \frac{3}{y} = 35$, $\frac{3}{x} - \frac{1}{y} = 3$

(v) $2\sqrt{x} + 3\sqrt{y} = 35$, $3\sqrt{x} - 2\sqrt{y} = 3$

11. The formula for the sum of the cubes of the integers

between 1 and n is $\frac{n^2}{4}(n+1)^2$

i.e. $1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \frac{n^2}{4}(n+1)^2$

So, for example, $1^3 + 2^3 + 3^3 + 4^3 = \frac{4^2}{4}(4+1)^2 = 100$.

(a) Use this formula to find the value of each of the following:

(i) $1^3 + 2^3 + 3^3 + \ldots\ldots + 20^3$

(ii) $1^3 + 2^3 + 3^3 + \ldots\ldots + 40^3$

(iii) $21^3 + 22^3 + 23^3 + \ldots\ldots + 40^3$

(iv) $10^3 + 20^3 + 30^3 + \ldots\ldots + 200^3$

(b) Given that $1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = 2,047,761$, find the value of n.

12. I have two dice. One is red and the other is blue. The red die has faces labelled £0, £0, £0, £1, £1, £1, and the blue die has faces labelled £0, £0, £0, £0, £1, £2. In a village fete you pay £1 to throw the dice, and the winnings is the sum of the values on the two dice.

(a) Calculate the probability you win £0.

(b) Calculate the probability you win £1.

(c) Calculate the probability you win £2.

(d) Calculate the probability you win £3.

Is the fete likely to make a profit from this game? Justify your answer.